Influence of phase noise due to air fluctuations in Terahertz Time-Domain Spectroscopy setups

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THz-TDS systems are low-noise systems whose precisions are not only limited by acquisition time but also by air fluctuations. We provide a methodology and perform measurements on two distinct THz-TDS system and we show that free-space optical path variations are dominant in spectral phase noise. The trade-off between integration-time and noise proves not to be straightforward, while a simple noise model for THz-TDS setups can discriminates different sources of noise. The relation between air fluctuations and spectral phase noise are discussed.

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Introduction

Terahertz Time Domain Spectroscopy (THs-TDS) systems have become more and more popular thanks to their ability to capture coherent information, their ambient temperature operation and stability (hence their commercial availability) and their low-noise figures compared to other THz-range techniques such as Bolometers or Quantum Cascade Lasers.

There are many sources for the noise in THz-TDS that have been investigated, such as the noise produced by photoswitch antenna themselves [1], laser beam deflection [2] or laser power fluctuations and methods to circumvent them [3]. Some other groups how studied how the noise itself modifies the data [4, 5] and how error propagates [6].

Noise definitions and experimental assessment are of major importance for two main reasons : evaluation of a system (*i.e.*, how far can I go with my setup ?) and comparison among different systems (*i.e.*, which system would be the best for the experiment I want to make ?). But adequacy of the many definitions for the noise depends on the inner functioning of the system under study.

Thus, these definitions must be handled with care. Naftaly et al. [7] proposed a full framework, using two common metrics for the noise in a system : Signal-to-Noise Ratio (SNR) and Dynamic Range (DR). We propose to extend their work and suggest a noise model, that allows to separate and compare the influence of various sources of noise. This is important because relatively low acquisition speed of THz-TDS systems due to point-to-point scanning are compensated by the tremendous noise-filtering effect of the lock-in amplifiers (LIA). But since THz-TDS systems can be thought as complex interferometric setups, and because the phase-retrieval is a main feature of THz-TDS, issues such as phase noise and air fluctuations that ultimately limit the precision must be investigated.

1. Definitions for the noise and experimental assessment

We proceeded the measurements on typical THz-TDS setups with photoswitch antennas, which detailed description can be found elsewhere[1]. A femtosecond (fs) laser (76 MHz repetition rate) shine on the emission antennas, generating a THz pulse, modulated by an optical chopper at 280 Hz. The pulse is then detected after propagation by a photoswitch antenna and the signal is filtered with a Lock-In amplifier (LIA). The time-delay τ between the pump and the probe is provided by an optical delay-line, whose variation allows to reconstruct the THz waveform $s(\tau)$. We made use of two different THz-TDS setups to provide diversity in the results and avoid systematic errors, both located in the same room and fed by the same fs-laser. The first setup (referred to as 'S1') features a collimated beam propagating over a length 123 cm, while the second setup ('S2') is a typical TDS setup, with focused THz beam propagating over 97 cm in a sealed box filled with nitrogen to purge atmospheric water vapour. On both setups, emission and reception photo-switch antennas were provided by IEMN Technological platform from different batch. The fluctuations of the laser in intensity were about 1%, and the beam direction was stabilized by the mean of a piezoelectric-controlled mirror paired with a four-quadrants detector for closed-loop operation.

The measurements were made Palaiseau, France, during the month of July. The temperature of air was stabilized at 22 °C using an HVAC, the air blown being diffused and then dried by dehumidifier, leaving the relative humidity below 20%. All measurements were performed at the end of the day to ensure all the components have reached a steady-state temperature.

Conventions in this paper are the following : $s(\tau)$ is the (noiseless) THz waveform, $\tilde{S}(\nu)$ is the magnitude of its Fourier transform at frequency ν, $s(\tau, t)$ is the measured waveform performed at a time t. When the second variable is omitted, we refer to the signal with no noise. We will consider sets of measurements made at different times $\{s_k(\tau)\}_{k=1..N}$, all sampling points τ being measured in the same run.

The first noise metrics we define is the base noise σ_1, that can be measured for time-delays for which the THz pulse hasn't reached the detector yet (τ_0 is the start of the pulse).

$$\sigma_{1,n}^2 = \frac{1}{\tau_0} \int_{0<\tau<\tau_0} s_n^2(\tau) d\tau. \tag{1}$$

A quick estimate of the SNR of a system can be obtained by dividing the maximum in amplitude of the detected pulse by the standard deviation of this noise. That's usually[2] how the SNR is defined, but it doesn't take into account the measure-to-measure stability or the dependence of the noise on the sampling point τ.

To elude the latter definition shortcomings and take into account the pump-probe nature of THz-TDS experiment, it is natural to consider a definition where the variance $\sigma_T^2(\tau)$ of the noise between different measurements for every sampling point :

$$\sigma_T^2(\tau) = \frac{1}{N} \sum_k^N s_k^2(\tau) - \left(\frac{1}{N} \sum_k^N s_k(\tau) \right)^2 . \tag{2}$$

That's the definition proposed by Naftaly et al. [7] and it is adapted to THz-TDS experiments. However, their experiment and those we performed on both setups (figure **??**) show that the maximum standard deviation of the noise is not at the peak of the THz pulse (as one would expect) but instead where the slope is maximum *i.e.*, on both rising and falling edges. Moreover, noise level doesn't decrease uniformly when the integration time of the LIA in increased, as shown on figure **??** : when the derivative of the signal is high, the apparent noise level cannot be reduced further and actually increase, due to longer signal acquisition time (time between experimental run is kept constant).

[1] reference

[2] reference, definition commerciale

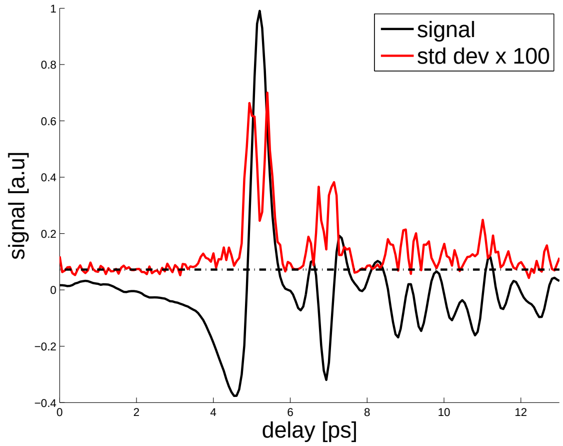

Fig. 1. Standard deviation of the noise $\sigma_T(\tau)$ at different sampling time-delays between 20 measurements performed on system S1, with an integration time of 10 ms per point. σ_1 noise is represented with dashed line. Notice that noise level is lower at the maximum of THz pulse than at the surrounding.

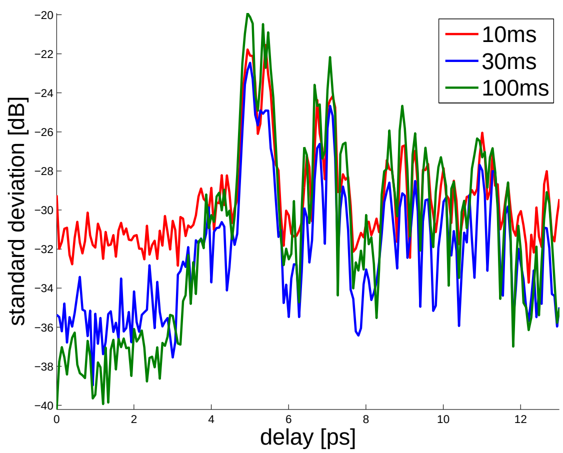

Fig. 2. Standard deviation of the noise $\sigma_T(\tau)$ at different sampling time-delays between 10 measurements, represented in a logarithmic scale for different LIA integration time. Notice that an increase in integration time has actually adverse effect.

One of the great advantage of THz-TDS system is the possibility to have access to the spectral content by the simple computation of the Fourier Transform of the time-domain signal. Even though the amplitude/time and the magnitude/phase representations bear the same information content, the noise has a notably different manifestation. If we consider, for example, that the noise is an additive white gaussian noise (AWGN) in the time domain, the influence of the noise σ on the real and the imaginary part of the Fourier Transform is independent, leading to a different manifestation for the noise in the frequency domain (see figure **??**).

Using the Fourier Transform of the signal, we can define three other metrics for the noise. Mimicking the definition for $\sigma_T(\tau)$, we can define the standard deviation $\sigma_\nu(\nu)$ of the spectral magnitude among the Fourier Transform of several measurements.

$$\sigma_\nu^2(\nu) = \frac{1}{N}\sum_k^N |(\tilde{S}_k(\nu)|^2 - \left(\frac{1}{N}\sum_k^N |(\tilde{S}_k(\nu)|\right)^2 . \tag{3}$$

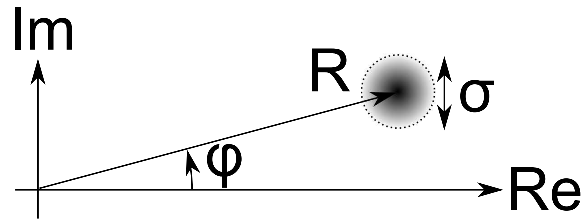

Fig. 3. An AWGN of variance σ^2 in the the time-domain translates to a Rayleigh distribution for the noise in spectral magnitude and a uniform distribution for the noise in in spectral phase.

We can see in figure **??** that there in no particular spectral features in the Fourier transform of the signal, in particular no $1/f$ noise due to electronics that generally arises, since we are dealing with a pump-probe scheme. The spectral density of the noise is rather constant, being a little lower in the absence of frequency

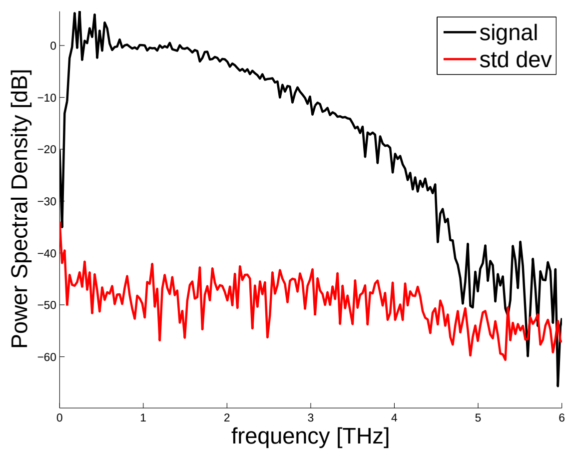

Fig. 4. Standard deviation $\sigma_\nu(\nu)$ of the spectral magnitude for different frequency ν, for pulses measured on system S2 with LIA integration time of 30 ms.

component in the signal, which allow to define a floor noise σ_2 which is the average of the spectral magnitude noise

$$\sigma_2^2 = \frac{1}{\nu_{\max}} \int_0^{\nu_{\max}} \sigma_\nu^2(\nu). \tag{4}$$

This definition provides a better rule of thumb estimate of the performance of a THz-TDS system than σ_1, for it takes into account the noise produced by the THz generation/detection process.

The spectral phase noise attracted very little attention, while spectral phase is as important as spectral magnitude for the characterisation of physical properties of materials. Since the spectral phase needs a reference, we can chose any measurement $\tilde{S}_n$ and then calculate for each the standard deviation of the phase $\sigma_{\phi,n}(\nu)$ difference between several measurements:

$$\sigma_{\phi,n}^2(\nu) = \frac{1}{N}\sum_k^N \arg[\tilde{S}_k(\nu)\tilde{S}_n^*(\nu)]^2 - \left(\frac{1}{N}\sum_k^N \arg[\tilde{S}_k(\nu)\tilde{S}_n^*(\nu)]\right)^2. \tag{5}$$

The spectral phase standard deviation shows a clear linear trend[3], as seen of figure **??**, which translates a frequency dependence that cannot be attributed a stationary noise.

[3]mettre un guide pour l'oeil

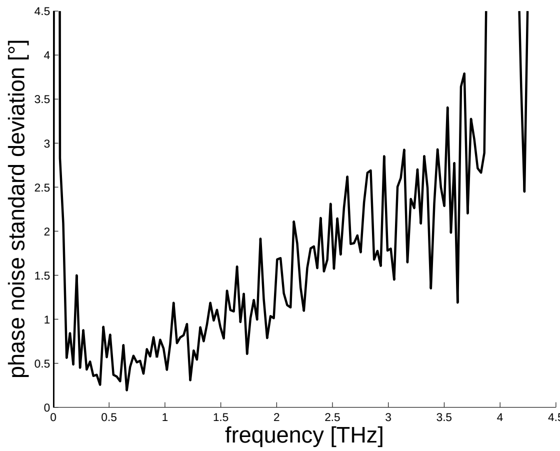

Fig. 5. Standard deviation $\sigma_{\phi,n}(\nu)$ of the spectral phase among different measurements performed on system S2 with LIA integration time of 30 ms. Notice the linear trend.

2. Noise model and critical evaluation

There are many sources of noise in a THz-TDS system that can be breakdown in at least four distinct categories. The first source of noise is the THz background noise, which is efficiently filtered by the coherent detection process and is therefore of no concern. The second source of noise $n_b(t)$ is the acquisition electronics noise which is responsible for the noise in obscurity and relates to σ_1; it reduces with an increase in LIA integration time. The third source of noise $n_s(t)$ is related to the THz generation and detection process itself, and comprises laser fluctuations in power and direction, vibrations of the optical elements and the quantum nature of photo-carriers generation in photoswitch antennas [1]. A fourth source of noise $n_\phi(t)$ a non-stationary phase noise (or jitter) due to the air fluctuations the optical path of the in either laser or the THz beam.

We hence propose a simple model noisy signal $s(\tau, t)$ that adds to the THz waveform $s(\tau)$ elements of the noise breakdown :

$$s_t(\tau, t) = s(\tau + n_\phi(t))\,[1 + n_s(t)] + n_b(t) \tag{6}$$

An adequate choice of particular sampling points τ in the terahertz pulses makes it possible to exalt or suppress one of the three noise component.

When the sampling point τ_{bg} is taken so that the THz hasn't reach the detector (or if we block the THz beam), we have $s(\tau_{\mathrm{bg}}) = 0$ and $\frac{\partial s}{\partial \tau}(\tau_{\mathrm{bg}}) = 0$, so that

$$s_t(\tau_{\mathrm{bg}}, t) = s(\tau_{\mathrm{bg}} + n_\phi(t))(1 + n_s(t)) + n_b(t) \simeq n_b(t), \tag{7}$$

what gives an estimate the noise linked with electronics $n_b(t)$.

At the peak of the THz pulse $\tau_{\max}$, where by definition the signal is maximum and the derivative is zero, we have :

$$s_t(\tau_{\max}, t) = s(\tau_{\max} + n_\phi(t))(1 + n_s(t)) + n_b(t) \simeq n_s(t)s(\tau_{\max}) + n_b(t) + s(\tau_{\max}), \tag{8}$$

what allows to evaluate $n_s(t)$, which is linked with the detection/generation process, if background noise is negligible, and limits the influence of jitter.

In the middle of the pulse τ_{inv} where the polarity of the pulse reverts , we have $s(\tau_{\mathrm{inv}}) = 0$ and $\frac{\partial s}{\partial \tau}(\tau_{\mathrm{inv}}) \neq 0$, hence :

$$s_t(\tau_{\mathrm{inv}}, t) = s(\tau_{\mathrm{inv}} + n_\phi(t))(1 + n_s(t)) + n_b(t) \sim n_\phi(t)\frac{\partial s}{\partial \tau}(\tau_{\mathrm{inv}})(1 + n_s(t)) + n_b(t) \simeq n_\phi(t)\frac{\partial s}{\partial \tau}(\tau_{\mathrm{inv}}) + n_b(t) \tag{9}$$

If $n_b(t)$ is sufficiently small, we can evaluate the influence of the phase noise $n_\phi(t)$.

Finally, a sampling point in the principal rising edge $\tau_{\rm sl}$ where all the noises combines, due to high amplitude and high derivative, for control.

The measurements where performed by setting the delay line at four sampling point previously defined, recording the LIA reading every 100 ms, increasing the LIA integration time (T_c) calibre every 12 minutes. The results are shown in figure **??**.

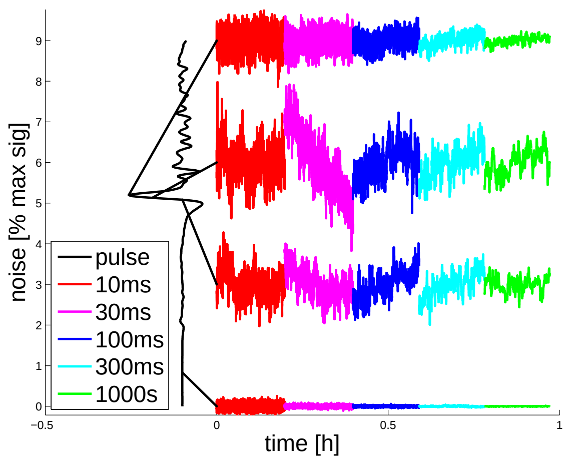

Fig. 6. Temporal evolution of the signal reading for different sampling points $\tau_{\rm bg}$, $\tau_{\rm inv}$, $\tau_{\rm sl}$ and $\tau_{\rm max}$ of the THz pulse (whose typical waveform is shown on the left). The LIA integration time calibre is progressively increased. Each is measurement is offsetted by 2% for clarity. Clear drift and/or influence of integration time is to be noted depending on the situation.

A relatively long-term drift is to be noted for non-zero derivative sampling points $\tau_{\rm sl}$ and $\tau_{\rm inv}$, which betrays the non-stationary behaviour of air fluctuations. Furthermore, autocorrelation function suggests indicates a typical fluctuation short-time scale of 10 seconds in this case. In opposition, there is very little drift and a sensible influence of the LIA T_c on the sampling points of zero derivative $\tau_{\rm bg}$ and $\tau_{\rm max}$. The signal-to-noise ratio as a function of T_c for sampling points $\tau_{\rm bg}$ and $\tau_{\rm max}$, is shown on figure **??**; the increase for $\tau_{\rm max}$ seems hampered by detection/generation noise n_s.

3. Air fluctuations and phase noise

The latter results show that except in particular sampling points, air fluctuations are the principal contributor in signal alteration. This is because the pump (generation, comprising the THz propagation) and the probe (detection) arms of the THz-TDS systems follow different free-space optical path experiencing uncorrelated fluctuations.

It is possible to give an estimate of the standard deviation of the differential optical path length. Let's consider that during a single pulse measurement s_k the perturbation of the optical path $(\Delta\tau)_k$ remains constant and that n_k accounts for all amplitude noise sources:

$$s_k(\tau) = s(\tau - (\Delta\tau)_k) + n_k(\tau). \tag{10}$$

The Fourier transform of such a signal writes :

$$\tilde{S}_k(\nu) = \tilde{S}(\nu) e^{\imath 2\pi\nu(\Delta\tau)_k} + \tilde{n}_k(\nu). \tag{11}$$

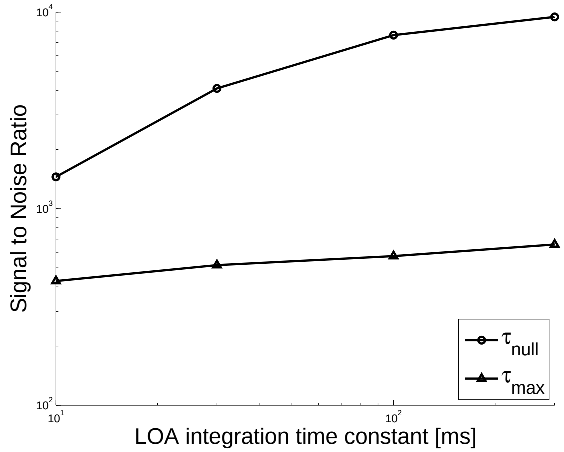

Fig. 7. Evolution of the signal-to-noise ration with an increase in LIA integration time, for two different sampling points $\tau_{\rm bg}$ and $\tau_{\rm max}$ on the terahertz pulse.

Hence the spectral phase difference between two measurements made at different time s_i and s_j is :

$$\begin{aligned}\Delta\phi_{i,j}(\nu) &= \arg[\tilde{S}_i(\nu)\tilde{S}_j^*(\nu)]\\ &= \arg\left[|\tilde{S}(\nu)|^2 e^{\imath 2\pi\nu((\Delta\tau)_i-(\Delta\tau)_j)}\left(1+\underbrace{\frac{\tilde{n}_i(\nu)\tilde{n}_j^*(\nu)e^{\imath\phi_1(\nu)}}{|\tilde{S}(\nu)|^2}+\frac{\tilde{n}_i(\nu)e^{\imath\phi_2(\nu)}}{\tilde{S}(\nu)}+\frac{\tilde{n}_j^*(\nu)e^{\imath\phi_3(\nu)}}{\tilde{S}(\nu)}}_{\text{perturbations}}\right)\right]\\ &= 2\pi\nu\left[(\Delta\tau)_i-(\Delta\tau)_j\right]+\phi_g(\nu)\end{aligned} \tag{12}$$

where $\phi_{\{1,2,3\}}(\nu)$ are spectral phases and $\phi_g(\nu)$ the compounded spectral phase that accounts for the noise related to amplitude fluctuations. The relative spectral phase hence bears a linear component, that is dominant in previous measurements (figure **??**). With equation 4, we can make a linear regression on the standard deviation of the phase for each frequency : $\sigma_{\phi,n}(\nu) = 2\pi\nu\sigma_L/c + o(L)$. The standard deviation of the optical path length on system S2 is then σ_L=625 nm (that amounts to a 2 fs time-delay, which is 3% of the 66 fs temporal sampling step).

This figure can also be estimated directly in the time domain, with the calculation of the derivative $\frac{\partial s}{\partial \tau}$ from measurement at high slope $s_t(\tau_{\rm inv})$ and $s_t(\tau_{\rm sl})$ (figure **??**). Using the finite expansion $s(\tau+\Delta\tau)-s(\tau) \sim \Delta\tau\frac{\partial s}{\partial \tau}$, we have :

$$\Delta\tau \sim \frac{s(\tau+\Delta\tau)-s(\tau)}{\frac{\partial s}{\partial\tau}(\tau)} \simeq \frac{\max_t\left(s(\tau,t)\right)-\min_t\left(s_t(\tau,t)\right)}{\frac{\partial s}{\partial\tau}(\tau)}. \tag{13}$$

The estimation of extremal variation of the optical path over an hour of measurement at sampling points $\tau_{\rm inv}$ and $\tau_{\rm inv}$ are 3.5 μm and 3.7 μm respectively, having a standard deviation of 628 nm and 545 nm respectively, what is consistent with the results found with the spectral phase method, and with fluctuation range experienced by other ultra-fast optics groups in the same building [8].

Discussion

We have tried to control all the elements that could have induced a bias in the experiments. First, the effect of laser fluctuations in power can be partly neglected at high level of signal, where air fluctuations influence is dominant. We cannot confirm the influence of laser direction fluctuation discussed by Takeda et al. [2], since we corrected the direction of the beam with a piezo-electric controlled mirror, but we considered that the effect of laser direction are equivalent to fluctuations in laser. Such fluctuations are much more sensible in systems using electro-optical detection [9] and should not be neglected.

There is no effect of laser deflection correction on the optical length, since it is placed well ahead of the beam-splitter separating generation and detection arm. Moreover, using fixed positions for delay-line confirm that phase difference between different measurements are not due to mechanical positioning uncertainty, that would be well bigger than manufacturer specifications. We were not able to assess performances of fibred THz-TDS systems, but they might be prone to thermal fluctuations leading to similar effects.

We used system S1 for discussion in time-related variations because the arms are larger (generation arm : 1.57 m, detection arm : 2.80 m), but we preferred the nitrogen-dried S2 (generation arm : 1.18 m, detection arm : 2.15 m) for discussion on phase, since to the absence of the vapour absorption lines eases the discussion. We double-checked on both setups all conclusions.

We can propose an alternative summation procedure that limits the influence of the optical path fluctuations in order to understand better how the noise figure evolves as a function of the amplitude in the time-domain. The principle is to zero the relative spectral phase between all measurements and a measurement s_n taken as a reference before summing them in the time domain (F^{-1} is the inverse Fourier Transform):

$$s_n^{\mathrm{moyen}} = \frac{1}{N}\sum_k^N F^{-1}\left(\tilde{S}_k \exp(-\imath \arg[\tilde{S}_k \tilde{S}_n^*]\right) = \frac{1}{N}\sum_k^N F^{-1}\left(|\tilde{S}_k| \exp(\imath \arg[\tilde{S}_n])\right). \tag{14}$$

Spurious fluctuations near maximum slope points tend to disappear (see figure **??**) while standard deviation of the noise concentrates at high level of signal. We find back the 1:4 ratio between base noise and signal

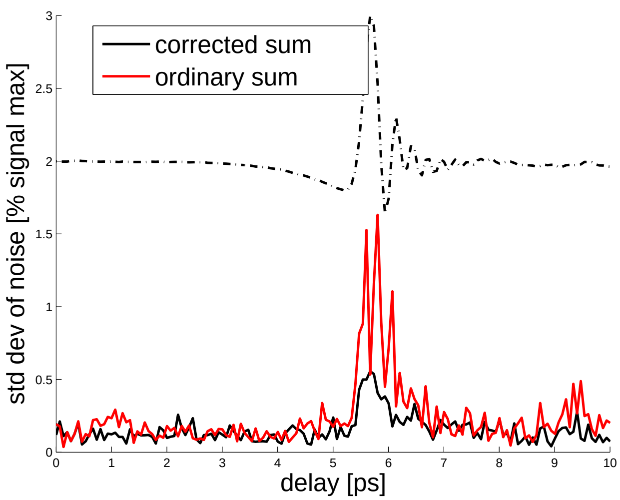

Fig. 8. Sommation de signaux corrigs en phase; impulsion THz pour comparaison.

noise witnessed in fixed delay-line experiment of figure **??**. This procedure doesn't correct the absolute phase fluctuation, for it imply the choice of phase reference on a measurement that is not corrected itself corrected in phase, but it helps in discriminating the detection/generation noise n_s. Still, we cannot determine the relative contributions of all noise sources at stake in n_s, for we cannot discriminate them.

Conclusion

The air fluctuations are a limiting factor in free-space THz-TDS experiments, and should be taken into account when designing experimental procedures. They can be efficiently reduced by protecting the laser and THz beams with plastic pipes. Moreover, an increase in LIA integration time and consequently in total acquisition time produce but smoother signals but does not necessarily improve accuracy. The use summation of many acquired signals should be performed with care, especially when they are performed over more than seconds. Fibred THz-TDS systems might be less sensitive to fluctuations, but this requires a further examination.

While time of acquisition and time-delay sampling are often confounded, this study give a methodology for assessing spectral phase precision (and not resolution), which is of particular importance since the direct retrieval of phase is a major feature in the THz domain, for the study of polarization or biological imaging [10, 11][4]. The proposed fixed sampling point method might also be used for the study the air fluctuations themselves, or in the presence of fumes, where conventional optics would be blocked by diffusion. Moreover, this study questions the ubiquitous use of constant sampling steps, and try to reveal features in hidden temporal information by moving to the frequency domain. A similar study using sparse coding[5] might reveal new features.

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[4]autopromo?
[5]trouver une bonne reference